Smoothing of the Gaussian contrast and parametric estimation for some degenerate ARCH processes

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When the volatility of an ARCH process is degenerate, the Gaussian contrast cannot be used for the parametric estimation of its coefficients. We will consider a typical example : the case of some ARCH processes with a linear volatility (LARCH) introduced to model the long memory ([3]). In the litterature, a weigthed least squares estimation ([2]) and a smoothed version of the (pseudo) gaussian likelihood ([1], [4]) have been proposed in order to estimate the coefficients at the usual rate $\sqrt{n}$ but the asymptotic efficiency of these methods has not been investigated. After giving an interpretation and a generalization of the smoothing method, we will show that under some general assumptions, it is possible to compute exactly the smallest asymptotic variance for LARCH processes. The result is atypical. The best asymptotic variance is a degenerate matrix of rank one, whose expression is related to the single "non-explosive" direction for an increasing approximation of the (pseudo) Fisher information. However, the choice of the smoothing parameter for a finite sample is an unsolved problem.
This method of smoothing can be easily extended to some multivariate ARCH processes or diffusions in order to recover some classical results of the non-degenerate case.

Références

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